18 Months Post-Doctoral Research Visit F/M

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Keywords. Scientific computing; Boltzmann equation; AP schemes; Multiscale methods.
Starting Date. December 2019 – January 2020.
Level of qualifications required. PhD or equivalent.
Instruction to apply. CV, motivation letter, list of publications, one or more letters of recommandation, and a short research statement.

Subject

Many engineering or biological problems involve fluid-like systems in transitional regimes: micro-electro-mechanical systems, space shuttle reentry, powder and grains, bacteria, *etc*. As a consequence of this nonequilibrium behavior, the usual fluid descriptions can break down, and a kinetic model may be needed to depict accurately the system. Such models are described by a particle distribution function $f^\varepsilon = f^\varepsilon(t,x,v)$, for $t \geq 0$, $x \in \mathbb{R}^{d_x}$, $d_x \leq 3$ and $v \in \mathbb{R}^3$, solution to the initial value problem

$$\begin{cases} \dfrac{\partial f^\varepsilon}{\partial t} + v \cdot \nabla_x f^\varepsilon = \dfrac{1}{\varepsilon} \mathcal{Q}(f^\varepsilon), \\ f^\varepsilon(0,x,v) = f_0(x,v) \geq 0, \end{cases} \tag{1}$$

where $\mathcal{Q}$ is the *collision* operator, describing a microscopic "collision process" [CIP94].

The development of easy-to-implement **Asymptotic Preserving (AP) schemes** [DP14] for kinetic equations of type (1) is a very important domain in multiscale modeling nowadays. Such a scheme needs to provide an **accurate** and **efficient** approximation of the general kinetic equation, while altogether being **stable** in the $\varepsilon \to 0$ limit, and providing an accurate approximation of the limiting problem **independently** on the time step or mesh size.

Impliciting in time is usually the correct way to design AP methods. Nevertheless, it is not always possible to adopt such strategies due to many different problems, such as the size of the set of considered ordinary differential equations (ODE), and the cost of solving large linear problems; the need to deal with a large – explicit – code and not re-engineer it completely; or nonlinearities embedded in the equation making inversion not possible.

A general time discretization technique called **Projective Integration** (PInt) was introduced in [GK03] to overcome some of these difficulties. It is designed to numerically solve systems of nonlinear and stiff ODEs, obtained *e.g.* as a semi-discretized form of a partial differential equation. PInt aims at mimicking the continuous behavior of this ODE system, namely that because of some stiffness, the solution u will be projected on a slow, low dimensional manifold in a very short time. The formal idea is then to take a small number of time steps of an explicit **inner integrator**, with a time scale corresponding to the fast rate of damping of the components of u living on the fast manifold. Then, a **forward extrapolation** is performed with a large time step, corresponding to the remaining components of u living on the slow manifold.

Such a strategy has been used with great success for linear kinetic equations in both the diffusive and hyperbolic scalings in a series of paper starting in [LS12]. The idea was then extended in [MRS19] to a more physical class of nonlinear collision operators. To do so, the seminal work [EP75] was used, where the structure

of the **spectrum** of the linear Boltzmann operator is established: it can be decomposed as a slow fluid part lying near zero, and a fast well separated microscopic part. This allowed the introduction a totally explicit and high order accurate time integrator for the Boltzmann equation.

Objective. This post-doctoral research position aim to tackle the more general case of the **multiple species** Boltzmann equation. Indeed, realistic models of the upper atmosphere require to couple up to 20 different kinetic equations in order to take into account the atmospheric chemistry ($O_2 - CO_2 - H_2 - CH_4$ recombinations). Mathematically, this makes the development of PInt method more difficult, because of the more intricated spectral structure. Nevertheless, the works [BD16] recently precised this structure, while [WZRZ15] developed a numerical method for efficiently computing the collision operators involved. Using these results along with the numerical method that we introduced in [MRS19], the development of a PInt method for this type of model is planned, paving the way to realistic simulations of **space shuttles** re-entry, generalizing the ones conducted in [DLNR18] in the case of the classical Boltzmann equation.

Context

The project will be conducted in the **Laboratoire Paul Painlevé** – UMR CNRS 8524 (LPP) of the Université de Lille, which is the host institution. This research laboratory is composed of 130 researchers covering almost the whole range of possible domains in mathematics. The post-doctorate researcher will join the ANEDP (PDE and Numerical Analysis) research team of the laboratory, which is composed of 20 tenured faculty in applied mathematics. Part of the research will also takes place inside the **Inria RAPSODI** research team, which focus on the development of high order, structure preserving numerical methods for dissipative systems such as (1).

References

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[MRS19] W. Melis, T. Rey, and G. Samaey. Projective and telescopic projective integration for the nonlinear BGK and Boltzmann equations. *to appear in SMAI J. Comput. Math.*, 2019.

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